Overview and Connection to Culminating Project

In this Learning Task, students will explore the concept of area and calculate the area of triangles and parallelograms. They will extend their understanding and find the area of composite figures using either decomposition or negative space strategies. Students will then calculate the area of their playground floor plan.

Learning Objectives

Students will be able to

- Calculate the area of triangles and parallelograms.
- Use decomposition, negative space, and/or other strategies to calculate the area of trapezoids and other composite figures.
- Justify why the area formulas for parallelograms and triangles always work.
- Use academic language to explain their mathematical ideas.

Driving Questions

- Why do professionals need to know how to calculate area while designing, building, and maintaining a playground?
- How do you break apart or decompose a complex figure into smaller figures in order to find the area?
- What strategies can you use to calculate the area of a complex figure?

Assessment

Check for Understanding • Areas of Polygons

Timeline

Lesson 1 • Parallelograms

Lesson 2 • Triangles

Lesson 3 • Convince a Friend, Convince a Skeptic

Lesson 4 • Composite Figures

Lesson 5 • Team Planning

Check for Understanding • Areas of Polygons (see Handouts and Assessments)

Language Support Strategies

Highlight the academic vocabulary used in this task. Briefly preview the terms orally and have students repeat them aloud and share what they know (definition or real-life connections).

Academic Vocabulary

- area
- composite shape
- decompose
- negative space
- parallelogram
- rectangle
- trapezoid
- triangle

Language of Instruction

- compare and contrast
- construct
- diagram
- find
- reflecting pool
- shortcut
- skeptic
- strategy
- table
- viable argument

Materials, Supplies, and Technology

- Bulk foam square color tiles (optional) or sticky notes
- Rulers
- Measuring tape
- Graph paper
- Scissors
- One copy to display of the four Parallelogram Shapes in the Real World images (see Handouts and Assessments)
- One copy to display of the four Triangle Shapes in the Real World images (see Handouts and Assessments)
- Copies of Check for Understanding • Areas of Polygons

LESSON 1

PARALLELOGRAMS

WARM-UP

What Is the Area of My Table?

- Give students square sticky notes (or other tiles) and a ruler. Have them predict the area of their table or desk. Have them measure the area of their table or desk using both sticky notes and inches and fill in the table in their Student Edition. Remind students that area is the amount of space a flat surface occupies and is measured using square units.
- Tell students to refer to and revise the Unit of Measurement Graphic Organizer from Learning Task 1 as needed.
- Ask students, "Why do you think it is important for schools and factories to know the area of your desk or table?"

Predict the area of your table if measured in:

Square sticky notes:

Answers will vary.

Square inches:

Answers will vary.

Square feet:

Answers will vary.

Describe the strategy you will use to find the area of your table.

Answers will vary.

Find the area of the table using sticky notes and the ruler. Complete the table below.

	Length	Width	Area
Sticky Notes			_____ square sticky notes
Inches			_____ square inches

STUDENT EDITION

- Have students share the strategies they used to find the area of their table. Address misconceptions.

Possible Strategies	Possible Misconceptions
1. Measure the length and width of the table in sticky notes. 2. Multiply the length times width to find the area in "square sticky notes."	Students might measure the side of a sticky note first, multiply the length and width (in sticky notes) to find the area of the table, and then multiply the area of the table times the **length** of the sticky note's **side** instead of the area of the sticky note. Example: Give students 3 × 3 inch sticky notes. Students might find that their table is 10 by 12 sticky notes. They would multiply to find that their table has an area of 120 square sticky notes. **They might multiply 120 by 3 to convert the area to square inches.** This is incorrect. They should multiply 120 by 9 (the amount of **square** inches in each sticky note).
1. Measure the side of a sticky note using inches. 2. Find the length and width of the table in sticky notes. 3. Multiply the length (in sticky notes) by the number of inches on each side of the sticky note. 4. Multiply the width (in sticky notes) by the number of inches on each side of the sticky note. 5. Multiply the length (in inches) times the width (in inches) to get the area of the table in square inches.	
1. Measure the side of a sticky note using inches. 2. Find the area of the sticky note in square inches by multiplying side times side. 3. Then, find the length and width of the table in sticky notes. 4. Multiply the length times width to find the area of the table in "square sticky notes." 5. Multiply the area of the table in "square sticky notes" by the area of a sticky note in square inches. This will give you the area of the table in square inches.	

PROJECT ACTIVITY

Area of Parallelograms

- Show students the four Parallelogram Shapes in the Real World images or other images of playground equipment that feature parallelograms. (See Handouts and Assessments.)
- Explain that when building, sometimes it is necessary to measure the area of figures that are not rectangles.
- Have students complete the activities in their Student Edition. To find the area, have students use scissors and try out different strategies, such as cutting off corners and rearranging to make rectangles, estimating by counting partial squares as half of a square unit, etc. Through this work, they should discover the formula for the area of a parallelogram. Make sure they have solidified the formula by the end of the class discussion.
- At the end of class, ask students to share and compare their strategies and the shortcuts/patterns they discovered. Have students show that every parallelogram can be rearranged to make a rectangle without changing the area. Make sure that students show the connections between the length and width of specific rectangles to the base and height of corresponding parallelograms.

LESSON 1 • PARALLELOGRAMS




NOTE

Students may not know the formula for finding the area of parallelograms until after this activity is finished.

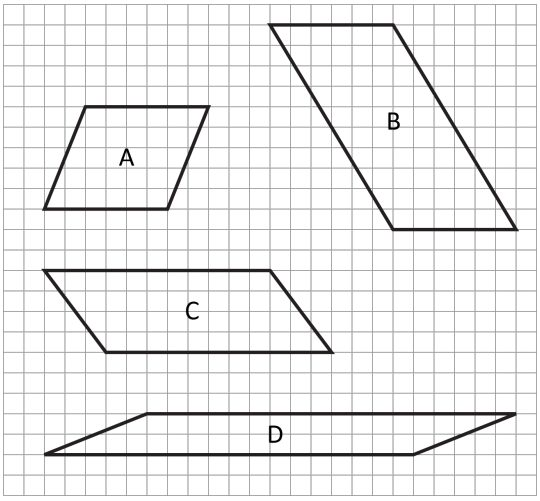
Compare and contrast rectangles and parallelograms.

Rectangles	**Parallelograms**
Must have all 90° or right angles. *Formula for finding area is length × width.*	*Can have 90° or right angles, but don't have to.*
Both *Both are quadrilaterals (four sides and four angles).* *Opposite sides are parallel.* *Opposite sides are congruent (same length).*	

STUDENT EDITION

LESSON 1 • PARALLELOGRAMS

Find the area of these parallelograms. Show your work.



Parallelogram A = 6 units × 5 units = 30 sq units
Parallelogram B = 6 units × 10 units = 60 sq units
Parallelogram C = 11 units × 4 units = 44 sq units
Parallelogram D= 18 units × 2 units = 36 sq units

Do you notice any shortcuts or patterns?

Answers will vary, but look for something like: You can always cut a triangle corner off and move it to the other side to make a rectangle.
Some students might notice that base × height always works.

STUDENT EDITION

Math Curricular Connection Suggestions

Khan Academy: Geometry

Khan Academy: Area of parallelograms

Learnzillion: Area

LESSON 2

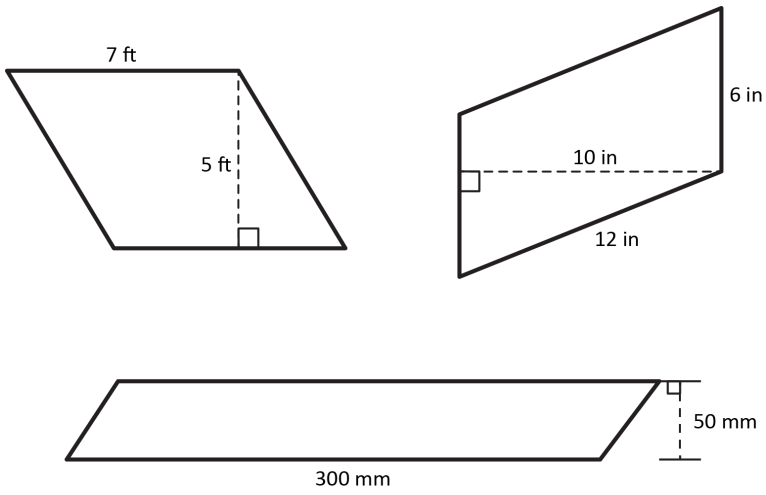
TRIANGLES

WARM-UP

Parallelograms

- Have students work on the warm-up activity in their Student Edition.

Find the area of these parallelograms. Show your work.



Area = 7 ft × 5 ft = 35 sq ft
Area = 6 in. × 10 in. = 60 sq in.
Area = 300 mm × 50 mm = 15,000 sq mm

STUDENT EDITION

- Remind students that all parallelograms can be rearranged to make rectangles. The base and height are what become the length and width. **The height is always *perpendicular* to the base.**

PROJECT ACTIVITY

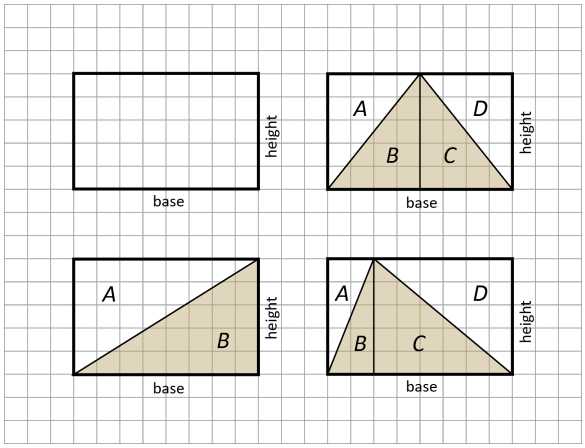
Area of Triangles

- Show students the four Triangle Shapes in the Real World images or other images of playground equipment that feature triangles. (See Handouts and Assessments.)
- Explain that when building, sometimes it is necessary to measure the area of figures that are not quadrilaterals.




- Have students complete the activities in their Student Edition.

Find the area of the rectangle and each of the triangles (*A*, *B*, *C*, *D*, and *B/C*). Show your work.



The rectangles have an area of 8 units × 5 units, or 40 square units.
In the figure below the rectangle, the two triangles are half of the rectangle, so they are 20 square units.
In the figure next to the rectangle, triangles A, B, C, and D are $\frac{1}{4}$ of the area of the rectangle, or 10 square units. Figures B and C together are 20 square units.
In the last figure, B and C together are 4 × 5, or 20 square units. Figures B and A are each $\frac{1}{2}$ of 2 × 5, or 5 square units each. That leaves 30 square units for triangles D and C together, or 15 square units each.

Do you notice any shortcuts or patterns?

The area of a triangle is always half of the area of the rectangle that encloses it. Triangles can look different, but if they have the same height and base, the area is the same.

STUDENT EDITION

- At the end of class, have students share and compare their strategies and the shortcuts/patterns they discovered. Make sure they show that every triangle has exactly one-half the area of the rectangle enclosing it.
- Challenge students to use a blank rectangle or graph paper to create a triangle with an area that is not half of the area of the rectangle that encloses it. When they discover obtuse triangles, have them **show that every rectangle has exactly one-half the area of a parallelogram**.

Math Curricular Connection Suggestions

Khan Academy: Geometry

Khan Academy: Area of triangles

Learnzillion: Area

LESSON 3

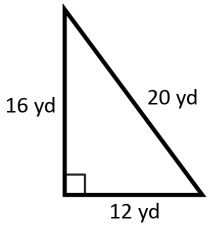
CONVINCE A FRIEND, CONVINCE A SKEPTIC

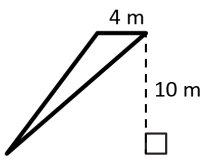
WARM-UP

Triangles

- Have students work on the warm-up activity in their Student Edition.

1. Find the area of these triangles. Show your work.

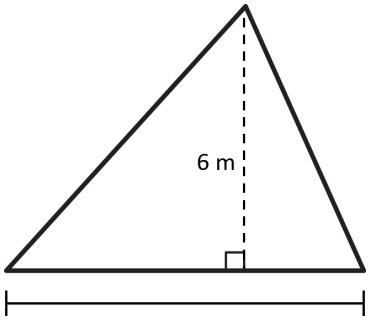




The right triangle has an area of $\frac{1}{2}$ bh, or $\frac{1}{2}$ • 12 yd • 16 yd, which equals 36 sq yd.

The obtuse triangle has an area of $\frac{1}{2}$ bh, or $\frac{1}{2}$ • 4 m • 10 m, which equals 20 sq m.

2. This triangle has an area of 30 m^2. What is the length of the base? Show your work.



$A = \frac{1}{2}\ bh$
$30 = \frac{1}{2}\ b \times 6$
$30 = 3b$
$10 = b$
The base is 10 meters.

STUDENT EDITION

PROJECT ACTIVITY

Convince a Friend, Convince a Skeptic

- Tell students this story:

  Jelani and Jameil want to build a shade cover over their playground, like the one in this image. Most of the pieces of their cover will be shaped like parallelograms or triangles. They need an efficient way of measuring the area of those figures so they know how much material they need to buy. They have asked for your help. Jelani knows how much you've been practicing math this year, so he will believe what you say as long as you explain it to him. However, Jameil is a skeptic. He will want you to prove that your explanation is correct.

CULTURAL CONNECTION

Playgrounds can get very hot or wet, depending on the weather. To help with this problem, covers are placed across the top to offer shade and shelter. Ask students to share with a partner a shade structure they have seen (e.g., palapas made of leaves in Mexico, Central and South America, and Hawaii).

- Instruct your students to complete the Convince a Friend, Convince a Skeptic activity in their Student Edition. Students can use drawings, notes, and figures from earlier lessons of this Learning Task.

Write a convincing argument to explain why the area formulas for triangles, parallelograms, and (extension) trapezoids work. Use words, numbers, and figures. Construct arguments for friends and skeptics.

For Friends	For Skeptics
• Explain **what** you know and **how** you know it is true. • Describe what you know using words, pictures, and numbers.	• Support your claims with evidence. • Justify your explanation. Explain **why** it is true. • Include examples. • Include examples that look like they might not work, but show that they actually do. • Describe what you know using words, pictures, and numbers.

1. The area of a parallelogram can always be found by using the formula $A = b$

2. $\bullet h$
(Area = base • height).

Convince a Friend	Convince a Skeptic

3. The area of a triangle can always be found by using the formula $A = \frac{1}{2} \bullet b \bullet h$
(Area = $\frac{1}{2}$ • base • height).

Convince a Friend	Convince a Skeptic

Extension

4. The area of a trapezoid can always be found by using the formula $\frac{1}{2} \bullet h(b_1 + b_2)$
(Area = $\frac{1}{2}$ • height • base 1 + base 2).

Convince a Friend	Convince a Skeptic

STUDENT EDITION

- At the end of class, have students share some of the arguments they created for Jelani and Jameil.

Math Curricular Connection Suggestions

Khan Academy: Geometry

Learnzillion: Area

LESSON 4

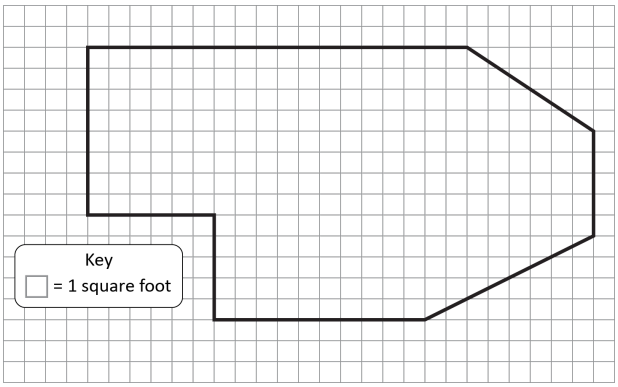
COMPOSITE FIGURES

WARM-UP

Playground Floor Plan

- Have students work on the warm-up activity in their Student Edition.

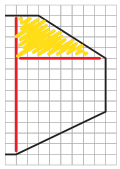
What is the area of the playground floor plan below?



See possible strategies below.

STUDENT EDITION

- Students will use a variety of strategies. It is important not to "force" students to use one prescribed method; their varying strategies will help drive a whole-class discussion.
- Circulate during the warm-up, paying close attention to the variety of strategies that students are using. Have students share and compare strategies for finding the area of the figure.

Possible Strategies	Possible Misconceptions
Decomposition: Break the shape into pieces of rectangles and triangles. Find the areas of those pieces and add them together to get the area of the total figure. (There are many variations of this strategy.)	Students might try to find the area of this piece as if it were a triangle, even though it is a trapezoid. 
Negative Space: Draw a rectangle enclosing the figure. Find the area of the whole rectangle. Find the area of the negative or unused space (rectangle in bottom-left corner and triangles in upper- and bottom-right corners). Subtract the area of the negative space from the area of the rectangle.	Students might just try multiplying two or more sides of the figure.

PROJECT ACTIVITY

What Is the Area of My Floor Plan?

- Have students calculate the area of their playground floor plan. Remind students that the reflecting pool cannot be built on or moved, so they need to show how they subtracted its area (negative space) or calculated around it (decomposition).

Show how your team calculated the total area of your playground floor plan. Remember, your reflecting pool can't be moved or built on, so it does not count as part of your area.

You must include:

- A diagram of your floor plan
- Words explaining your strategy and why you chose it
- A table showing your calculations

Make sure you label all your units of measurement.

Playground Floor Plan Name

Answers will vary by playground. See areas below.

Diagram

(Insert picture here or attach floor plan paper that your team marked.)

Strategy

Table

Diagram Piece	Type of Figure	Dimensions of This Figure	Area of This Figure

STUDENT EDITION

Here are the areas of the playground floor plans.

Areas of Playground Floor Plans

Playground Name	Total Area (in square yards)	Area of Reflecting Pool (in square yards)
Euclid	1,437	16
Hypatia	1,377	35
Aristotle	1,409	30
Noether	1,373	40
Pythagoras	1,377	45
Ramanujan	1,403	76
Gauss	1,362	52
Descartes	1,379	21
Euler	1,360.5	44
Pascal	1,427	24

STUDENT EDITION

Math Curricular Connection Suggestions

Khan Academy: Geometry

Learnzillion: Area

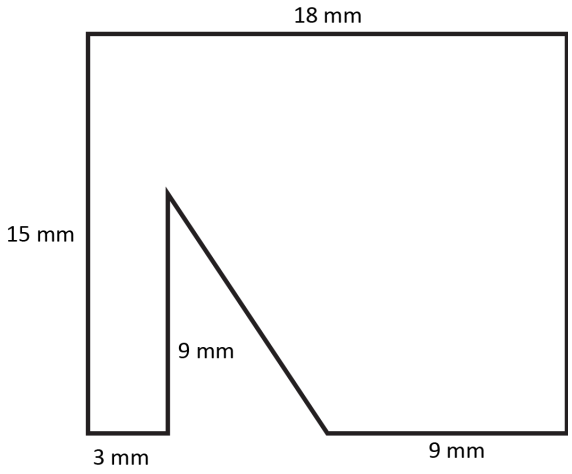
LESSON 5
TEAM PLANNING

WARM-UP

Composite Figure

- Have students work on the warm-up activity in their Student Edition.

A right triangle has been removed from the rectangle below. What is the area of the remaining figure?



The figure has an area of 243 square millimeters.

What strategy did you use to find the area? Why did you choose that strategy?

Answers will vary, but students might say:
I calculated the area of the whole figure: 15 × 18 = 270 sq mm.

Then I calculated the area of the triangle cutout: $\frac{1}{2} \cdot 6 \cdot 9 = 27$ sq mm.

Then I subtracted the two numbers: 270 – 27 = 243 sq mm.

I chose that strategy because it seemed like the least number of steps.

STUDENT EDITION

PROJECT ACTIVITY

What Is the Area of My Floor Plan?

- Give students more time to complete the "What Is the Area of My Floor Plan?" pages in their Student Edition and add more information or features to their Playground Information Table.

CHECK FOR UNDERSTANDING

Areas of Polygons

- Distribute the Learning Task 2 assessment: Check for Understanding • Areas of Polygons.

Select two of the figures shown.

Use a diagram or words to describe how to calculate the area of each figure you choose. Then find the area.

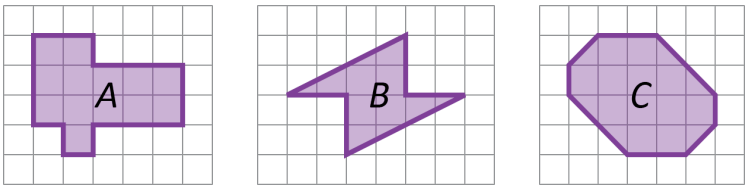
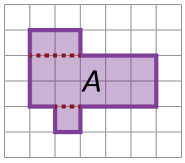
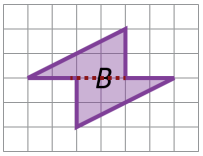
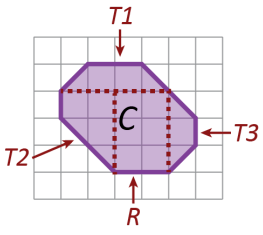


Figure (*A*, *B*, or *C*)	Describe how to calculate the area.	Area
Figure A		*1 unit² + 10 unit² + 2 unit² = 13 unit²*
Figure B		*Triangle area = $\frac{1}{2}$ • 4 • 2* *Triangle area = 4 unit²* *2 triangles = 8 unit²*
Figure C		*R = 2 • 3 = 6* *T1 = $\frac{1}{2}$ • 1(4 + 2) = 3* *T2 = $\frac{1}{2}$ • 2(3 + 1) = 4* *T3 = $\frac{1}{2}$ • 1(3 + 1) = 2* *Total area = 6 + 3 + 4 + 2 = 15 unit²*