**Subject area/course**: Mathematics/Algebra I or II

**Grade level/band**: 9-­‐12

**Task source**: Stanford University School Redesign Network and The Ohio Department of Education

**Open for Business**

**TEACHER'S GUIDE**

**A. Task overview**:

The context for this task immerses students in the mathematics of supply and demand. Accordingly, the work involves applying their understanding of demand, revenue, profit, and price. Students learn about how to model the relationships among these quantities using Internet resources provided for this purpose.

**B. Aligned standards:**

**1. Primary Common Core State Standards**

CCSS.Math.Content.HSA-­‐CED.A.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*.

CCSS.Math.Content.HSA-­‐CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods*.CCSS.Math.Content.HSA-­‐SSE.A.1b Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret P(1+r)n as the product of P and a factor not depending on P*.

CCSS.Math.Content.HSA-­‐SSE.A.2 Use the structure of an expression to identify ways to rewrite it. *For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2 + y2)*.

CCSS.Math.Practice.MP4Model with mathematics.

CCSS.Math.Practice.MP8Look for and express regularity in repeated reasoning.

**2. Secondary Common Core State Standards (optional)**

CCSS.Math.Content.HSA-­‐SSE.A.1 Interpret expressions that represent a quantity in

terms of its context.

CCSS.Math.Content.HSA-­‐SSE.A.1a Interpret parts of an expression, such as terms,

factors, and coefficients.

CCSS.Math.Practice.MP1Make sense of problems and persevere in solving them. CCSS.Math.Practice.MP3Construct viable arguments and critique the reasoning of others.

CCSS.Math.Practice.MP5Use appropriate tools strategically.

**3. Critical abilities**

Research: Conduct sustained research projects to answer a question (including a self-­‐ generated question) or solve a problem, narrow or broaden the inquiry when appropriate, and demonstrate understanding of the subject under investigation. Gather relevant information from multiple authoritative print and digital sources, use advanced searches effectively, and assess the strengths and limitations of each source in terms of the specific task, purpose, and audience.

Analysis of Information: Integrate and synthesize multiple sources of information (e.g., texts, experiments, simulations) presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to address a question, make informed decisions, understand a process, phenomenon, or concept, and solve problems while evaluating the credibility and accuracy of each source and noting any discrepancies among the data.

Communication in Many Forms: Use oral and written communication skills to learn, evaluate, and express ideas for a range of tasks, purposes, and audiences. Develop and strengthen writing as needed by planning, revising, editing, and rewriting while considering the audience.

Modeling, Design, and Problem Solving: Use quantitative reasoning to solve problems

arising in everyday life, society, and the workplace, e.g., to plan a school event or analyze a problem in the community, to solve a design problem or to examine relationships among quantities of interest. Plan solution pathways, monitoring and evaluating progress and changing course if necessary, and find relevant external resources, such as experimental and modeling tools, to solve problems. Interpret and evaluate results in the context of the situation and improve the model or design as needed.

**4. Other standards**

21st Century skills:

• Financial, Economic, Business and Entrepreneurial Literacy

• Communication and Collaboration

• Critical Thinking and Problem Solving

**C. Time/schedule requirements:**

Students should be given one week to complete the task. If you teach in a block schedule

you may have time each day to engage in instruction related to the task in addition to allowing students time for this performance task.

**D. Materials/resources:**

Access to computers with spreadsheet programs or access to graphing calculators.

**E. Prior knowledge:**

Students should be familiar with the concepts of supply, demand, revenue, profit,

wholesale price, retail price, and efficiency. Students also need an understanding of percents and taxes.

Prior grades’ knowledge of CCSS includes:

• 8.F.1. Define, evaluate, and compare functions. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

• 8.F.2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

• 8.F.4. Use functions to model relationships between quantities. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x*,

*y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

• 8.F.5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

**F. Connection to curriculum:**

This performance task is an advanced algebra task for high school students, as part of an

Algebra II course or in Algebra I following the study of quadratic functions. It includes the CCSS standards listed in section B above and may be used with students to apply their understanding of linear and quadratic functions. This task works well when students are learning to apply their understanding of quadratic functions. Ideally, students would not be told nor cued (by including it in a quadratics unit) that they will use their knowledge of quadratic functions and modeling with functions. For students to engage in and improve their higher order thinking skills and critical abilities, students should be given the freedom to devise their own solution strategy and decide which math tools are most appropriate, effective, and efficient to solve the task.

**G. Teacher instructions:**

When introducing this task, begin with a discussion of the relationship between price and

demand, highlighting the assumption that the relationship is linear. Use one item and various prices to develop a consensus that as the price goes up the demand for an item will go down. Plot the two data points for that item, and use a dotted line to illustrate the assumption that the demand function is linear. Note that the slope of the demand function is negative. (You may also do this with a different data set than that included in this performance task in order to allow students to apply their knowledge to the scenario about Malena.)

You need not remind students (as a whole class) how to find the equation of a line. Instead, let them get started and provide individual help to students who get stuck.

[One method of solving this problem is to develop the equations *Profit = Revenue – Cost* and *Revenue = pq* (price times quantity). You may ask the class how Malena will calculate her profit for an item. Also give students an opportunity to decide that *Revenue = pq* (price times quantity). Allow students to investigate on their own before discussing this with them.]

Keep in mind that the demand function is an approximation. Therefore, exact answers are not important. The demand quantities should be whole numbers, of course, so some rounding or truncation will be necessary.

Encourage students to collaborate on this task, to check both their thinking and their calculations. When first introducing the task, consider devoting an entire class period to working both individually and collaboratively.

**H. Student support:**

• Students may have trouble computing the slope of the demand equation and may compute the reciprocal of the desired slope. Because we are describing how demand depends upon price, the difference in *q*-­‐values is on top and the difference in *p*-­‐values is on the bottom. But correcting students before they have an opportunity to discover such an error would cut this learning process short.

• Encourage students to use graphing calculators or spreadsheets for the computations. With these technologies, it is not necessary to simplify the expressions and equations. It is sufficient to use a graphing calculator or a spreadsheet to find the maximum profit for an item.

**I. Extensions or variations:**

• In addition to the letter students can give a presentation of their reasoning to their peers as a class presentation or to a committee of teachers and students. Students may do presentations individually or in pairs or groups.

• An additional set of data can be provided to ask students to provide the same analysis and to allow for student choice of which data set they wish to analyze.

• The goal of $5,000 can be changed to a different value.

• The scenario of Malena raising money to tour South America can be changed. For example, the scenario could be that a student is trying to raise money to help support his or her family or to purchase a present for a family member, etc.

**J. Scoring:**

Student work can be scored using the Math Performance Assessment Rubric (Grades 9–12).

**Possible** **solutions**

*Supply and demand*

Demand is assumed to be equal to the number of a particular item sold, and is represented by *q*

in this discussion. Revenue is given by the equation *R* = *pq*, where *p* is the price of the item and *q* is the demand. Demand, assuming it is linear as specified in the task, is given by the equation *q = mp + b*, where *m* and *b* can be determined by using the numbers of items sold at different prices.

*Using tables and graphs to find a maximum value*

There are several approaches to finding the price that will maximize Malena’s profits. Using tables and graphing the equations are two good approaches. Both approaches provide important opportunities for self-­‐correction of errors along the way during the procedures described above. Graphing is especially useful for answering the larger question of the task about what products she should sell because graphs allow for more direct visual comparison among the outcomes for different products.

Finding the *p*-­‐value that maximizes profit, *P*, can also be done using Calculus by taking the derivative of the equation for profit. Another approach (equivalent to taking the derivative in this case) is to use the parameters of the quadratic equation for profits to identify the *p*-­‐value that maximizes *P*.

Each of these alternatives serves as a way of checking the conclusions drawn from work done with tables and graphs, but should not be used to the exclusion of tables and graphs. Note that the task specifies that graphs are required to provide a deeper glimpse into the mathematical behavior of the quantities in this situation.

*Mathematical modeling*

In modeling the relationships among quantities in this task, students are identifying the numbers of items that are *likely* to sell at certain prices, according to a linear demand equation.

This task requires developing models for at least two aspects of the situation in this task:

• The demand for different products in order to identify the profits that are possible for each (a linear equation).

• The profit Malena makes by selling products at different prices, accounting for the demand at that price, and accounting for the percentage taken by eBay (a quadratic equation).

Using these equations to model quantitative relationships is essential to the decision-­‐making process about which products should be sold, and for how much, in order for Malena to earn

$5000 within the first month.

For each item, the process is as follows: construct the demand equation, use the demand equation to express the profit in terms of the price, and then find the price that maximizes the profit.

Beginning with laptops, 15 were sold at a price of $900 and 12 were sold at a price of $950. The slope of the demand for laptops is computed as follows:

$$\frac{12-15}{950-900}= -\frac{3}{50}= -0.06$$

 Thus, q = −0.06p + b. To find the intercept, substitute the values 15 and $900:

15 = −0.06(900) + b = −54 + b b = 69

So the demand equation is q = −0.06p + 69 . Then revenue is given by R = pq .

Malena’s costs depend on both the wholesale price and the percentage taken by eBay. Because the price of the laptop is above $25, the eBay charge, *c*, is given by:

c = [(0.0875)(25) + (0.035)(p − 25)]

= (0.035p + 1.3125)

Then profit, *P*, which equals revenue minus total cost, is given by:

P = pq − 700q - (0.035p + 1.3125)q

= [p − 700 - (0.035p + 1.3125)]q

= [p − 700 - (0.035p + 1.3125)](− 0.06p + 69)

where *P* (profit), and *q* (demand), are given in terms of *p* (price) in the equations above.

Performing the necessary substitutions to express Malena’s profits exclusively in terms of *p* requires both attention and persistence. With these substitutions, her profits can be expressed as:

P = [p − 700 - (0.035p + 1.3125)](− 0.06p + 69)

= −0.0579p 2 + 108.664p − 48,390.6

Using a spreadsheet and the profit formula, here are the projected profits at various prices:

|  |  |  |
| --- | --- | --- |
| **Price** | **Demand** | **Profit** |
| 900 | 15 | $ 2,508.00 |
| 910 | 14.4 | $ 2,546.65 |
| 920 | 13.8 | $ 2,573.72 |
| 930 | 13.2 | $ 2,589.21 |
| 940 | 12.6 | $ 2,593.12 |
| 950 | 12 | $ 2,585.45 |
| 960 | 11.4 | $ 2,566.20 |

The maximum appears to occur at a price between $930 and $940. Looking more closely, here are the projected profits between those prices:

|  |  |  |
| --- | --- | --- |
| **Price** | **Demand** | **Profit** |
| 931 | 13.14 | $ 2,590.12 |
| 932 | 13.08 | $ 2,590.92 |
| 933 | 13.02 | $ 2,591.60 |
| 934 | 12.96 | $ 2,592.16 |
| 935 | 12.9 | $ 2,592.61 |
| 936 | 12.84 | $ 2,592.95 |
| 937 | 12.78 | $ 2,593.16 |
| 938 | 12.72 | $ 2,593.26 |
| 939 | 12.66 | $ 2,593.25 |

The price that will maximize profits on laptops is approximately $938, but the price that most closely gives an integer demand value is $933. The table below summarizes the results:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Item** | **Demand Equation** | **Price** | **Demand** | **Revenue** | **Wholesale****Costs** | **e-­‐Bay fees** | **Profit** |
| iPod | *q* = -­‐0.32*p* + 99 | $234 | 24 | $5,625 | $3,600 | $228 | $1,797 |
| X-­‐Box 360 | *q* = -­‐0.48*p* + 185 | $323 | 30 | $9,688 | $7,500 | $378 | $1,809 |
| Laptop | *q* =-­‐.06*p* + 69 | $933 | 13 | $12,133 | $9,100 | $442 | $2,592 |
| Stereo | *q* = 0.737*p* + 131.53 | $154 | 18 | $2,773 | $2,250 | $121 | $403 |
| Calculator | *q* = -­‐1.4*p* + 150 | $88 | 27 | $2,372 | $1,755 | $118 | $499 |

As can be seen in the table, laptops are the most profitable item, followed by XBoxs and iPods. To raise the $5,000 necessary Malena can sell 13 laptops, 30 Xboxes, and 8 iPods.