**Subject area/course**: Mathematics/Calculus

**Grade level/band**: 11-12

**Task source**: Summit Public Schools

**Optimization Project**

**TEACHER'S GUIDE**

1. **Task overview**:

In this project, students will use calculus to optimize a product or design. They will have to carefully characterize the variables and constraints of their system to form an equation to be optimized, and then take the derivative to find the maximum or minimum of that function. They must include precision justification/explanations for their initial setup, for each step of the solution, and for their analysis of the problem. Once the optimized measurements are calculated, they will create a physical model to match their calculations.

1. **Aligned standards:**
2. **Primary Common Core State Standards**

[CCSS.MATH.CONTENT.HSA.CED.A.2](http://www.corestandards.org/Math/Content/HSA/CED/A/2/) Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

[CCSS.MATH.CONTENT.HSF.BF.A.1](http://www.corestandards.org/Math/Content/HSF/BF/A/1/) Write a function that describes a relationship between two quantities.\*

[CCSS.MATH.CONTENT.HSF.BF.A.1.A](http://www.corestandards.org/Math/Content/HSF/BF/A/1/a/) Determine an explicit expression, a recursive process, or steps for calculation from a context.

[CCSS.MATH.PRACTICE.MP4](http://www.corestandards.org/Math/Practice/MP4/) Model with mathematics.

1. **Critical abilities**

Analysis of Information:Integrate and synthesize multiple sources of information (e.g., texts, experiments, simulations) presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to address a question, make informed decisions, understand a process, phenomenon, or concept, and solve problems while evaluating the credibility and accuracy of each source and noting any discrepancies among the data.

Communication in Many Forms:Use oral and written communication skills to learn, evaluate, and express ideas for a range of tasks, purposes, and audiences. Develop and strengthen writing as needed by planning, revising, editing, and rewriting while considering the audience.

Modeling, Design, and Problem Solving**:** Use quantitative reasoning to solve problems arising in everyday life, society, and the workplace, e.g., to plan a school event or analyze a problem in the community, to solve a design problem or to examine relationships among quantities of interest. Plan solution pathways, monitoring and evaluating progress and changing course if necessary, and find relevant external resources, such as experimental and modeling tools, to solve problems. Interpret and evaluate results in the context of the situation and improve the model or design as needed.

1. **Time/schedule requirements:**

This task will take approximately 2-3 weeks, depending on your school schedule and other activities.

1. **Materials/resources:**

Documents:

* Item A. Optimization Waffle Cone
* Item B. Optimization Cereal Boxes
* Item C. Proposal Outline
* Item D. Action Plan Guidelines
* Item E. Analysis Guidelines
* Item F. Final Product and Presentation Guidelines
* Item G. Bloom’s Project Reflection
1. **Prior knowledge:**

None listed.

1. **Connection to curriculum:**

None listed.

1. **Teacher instructions:**
2. Entry Event

Students will get a circle of paper and should be instructed to cut a radius in that paper so that the newly cut ends can overlap and create a cone. This cone can then hold the chocolate chips as they eat during the rest of the class.

Students will work in pairs and share the chocolate chips that they receive. Obviously they want to create a cone with the largest possible volume to maximize the number of chocolate chips they get. Once they are ready, they can tape their cone so that it stays closed. Before receiving their chocolate chips, they should trace the rim of their cone on a piece of paper and measure the radius. When they ask for their chocolate chips, they must explain their justification for why their design should maximize the volume of the cone/number of chocolate chips they will receive.

Chocolate chips should be counted before consumed. Once the class finished this activity, they will report their radius measurements and amount of chocolate chips that they received. This data can be recorded in a Google Spreadsheet and a graph should be displayed.

Discussion Questions:

* Who received the most chocolate chips?
* Was the radius large, small, or somewhere in between?
* How can we figure out the exact measurement to get the most chocolate chips?

This activity can then transition into the waffle cone introductory activity to introduce the calculus analysis for optimization.

1. Introductory problems
* There are two practice optimization problems (Items A & B) to highlight the process before students choose their own topic.
* These problems will give them an example to help them diagram their problem and identify the relationships between components of the system and preview the steps in their action plan.
1. Choose a topic
* While a list of ideas will be included (without communication of variables/relationship between variables), students are encouraged to pick a real world topic that interests them.
1. Proposal (Item C)
* Diagram: Student must include a diagram that labels all relevant variables and constraints
* Optimization equation: Students must use the constraints of the problem to develop a single equation to be optimized
1. Action Plan and Calculations (Item D)
* All steps of the calculation (taking the derivative, finding the critical points, finding the maxima/minima) must be explained/justified
* Students will have the opportunity to get their calculations reviewed by another student to check for calculation errors and review the process
1. Analysis (Item E)
* Graphs of optimization equation and its derivative must be included, along with an explanation of how the graph shows the optimum value for the problem.
1. Model Creation (Item F)
* Students will make a model based on calculated measurements. The model can be scaled down and made from simpler alternative materials as needed.
1. Presentation/Peer Feedback
* Students will present in smaller peer feedback groups. They will explain their process (without going step by step into the calculations) and justify their end result. All students in the group will have the opportunity to provide feedback that the presenter can respond to in the reflection.
1. Reflection (Item G)
* Students complete full class individual reflection based on different Bloom’s levels.
1. **Student support:**

None listed.

1. **Extensions or variations:**

None listed.

1. **Scoring:**

Student work can be scored using the Summit Public Schools Optimization Project rubric.