**Subject area/course**: Mathematics/Pre-Calculus

**Grade level/band**: 10-12

**Task source**: Envision Schools; Author: Cliff Cheng

**Projectile Motion**

**TEACHER'S GUIDE**

1. **Task overview**:

Students will create a math model to predict the path of a projectile. They will be required to complete and submit a formal paper of their problem solving process and solution for their scenario.

1. **Aligned standards:**
2. **Primary Common Core State Standards**

[CCSS.MATH.CONTENT.HSF.TF.B.7](http://www.corestandards.org/Math/Content/HSF/TF/B/7/) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.\*

CCSS.MATH.CONTENT.HSA.CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

1. **Secondary Common Core State Standards**

[CCSS.MATH.CONTENT.HSA.CED.A.4](http://www.corestandards.org/Math/Content/HSA/CED/A/4/) Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations

[CCSS.MATH.CONTENT.HSN.Q.A.1](http://www.corestandards.org/Math/Content/HSN/Q/A/1/) Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

1. **Critical abilities:**

Research: Conduct sustained research projects to answer a question (including a self-generated question) or solve a problem, narrow or broaden the inquiry when appropriate, and demonstrate understanding of the subject under investigation. Gather relevant information from multiple authoritative print and digital sources, use advanced searches effectively, and assess the strengths and limitations of each source in terms of the specific task, purpose, and audience.

Analysis of Information**:** Integrate and synthesize multiple sources of information (e.g., texts, experiments, simulations) presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to address a question, make informed decisions, understand a process, phenomenon, or concept, and solve problems while evaluating the credibility and accuracy of each source and noting any discrepancies among the data.

Communication in Many Forms**:** Use oral and written communication skills to learn, evaluate, and express ideas for a range of tasks, purposes, and audiences. Develop and strengthen writing as needed by planning, revising, editing, and rewriting while considering the audience.

Use of Technology**:** Present information, findings, and supporting evidence, making strategic use of digital media and visual displays to enhance understanding. Use technology, including the Internet, to research, produce, publish, and update individual or shared products in response to ongoing feedback, including new arguments or information.

Interpersonal Interaction and Collaboration**:** Develop a range of interpersonal skills, including the ability to work with others, to participate effectively in a range of conversations and collaborations.

Modeling, Design, and Problem Solving**:** Use quantitative reasoning to solve problems arising in everyday life, society, and the workplace, e.g., to plan a school event or analyze a problem in the community, to solve a design problem or to examine relationships among quantities of interest. Plan solution pathways, monitoring and evaluating progress and changing course if necessary, and find relevant external resources, such as experimental and modeling tools, to solve problems. Interpret and evaluate results in the context of the situation and improve the model or design as needed.

1. **Time/schedule requirements:**

This task will take approximately 10 days. Suggested timeline:

* Day 1: Discuss the project, set clear expectations regarding final product, and highlight the scenarios
* Days 2-3: Group work in class
* Days 4-5: Individual and/or group work outside of class
* Day 6: Review expectations of final project and peer edit
* Days 7-8: Individual and/or group work outside of class
* Day 9: Peer edits
* Day 10: Presentation of final product (Short summary of paper)
1. **Materials/resources:**
* Computers to access the Internet and to create final product
* Scientific calculator
* Paper, pencil, and graph paper (optional)
1. **Prior knowledge:**

Students should have an understanding of:

* linear and quadratic functions
* definitions of trigonometric functions in the context of right triangles and the unit circle
* inverse trigonometric functions and trigonometric identities
1. **Connection to curriculum:**

It is strongly suggested that this task follow an approximately weeklong instructional sequence on projectiles. This task would fit well within a unit on projectile motion, modeling, and/or analyzing solutions.

1. **Teacher instructions:**

This task begins with a week of in-class explorations to learn how to work through the component parts of a projectile motion scenario. Then, students will have a group to work with to investigate their particular prompt to start their projectile motion scenario.

1. **Student support:**
* Suggest using an online converter for students who struggle with converting units or rates.
* Provide students with an outline for the final product. Suggested outline:
	+ - Paragraph 1: Describe the scenario and why you find it interesting
		- Paragraph 2: Explain your plan/ approach. Explain what is needed to solve for the angle of the projectile.
		- Paragraph 3: What research did you complete? How reliable is the velocity you found? Provide citations to support your decision regarding the velocity you use.
		- Paragraph 4: Provide the solution. Explain how you arrived at the solution and what the solution means within the scenario.
		- Paragraph 5: Summarize the scenario and the solution. Mention any limitations of the process you used or the solution process itself.
* Formula that can be used:
	+ $R=\frac{v\_{0}^{2}sin2∅}{g}$
		- $R=range$
		- $v\_{0}=initial velocity$
		- $g$ = force of gravity
		- $∅=angle$
* Encourage struggling students to work on either scenario 3 or 4; the facts are more readily available and consistent.
* Challenge advanced students to provide a range of acceptable answers similar to those provided in Section J.
1. **Extensions or variations:**
* Ask students to examine the range of the initial velocity and the angle of the projectile. Students report the limitations of the scenario. Provide the maximum and minimum for the velocity (based on research) and the angle of the projectile. Determine the range for the horizontal distance the projectile can travel.
1. **Scoring:**

Student work can be scored using the SCALE Mathematics Performance Assessment Rubric, Grades 9-12.

Solutions

Scenario 1: Solutions: Professional quarterbacks can throw between 55 and 60 mph. The video shows the quarterback throwing between 68 and 68.3 yards, due to it being a diagonal pass and not directly in front of the quarterback. The pass takes approximately 2 seconds based on the play clock (not very reliable). The solution is that the quarterback had to release the ball at a 37.8◦ angle at 57 mph or at a 29.17◦ angle at 60 mph. All other solutions within this range are acceptable.

Scenario 2: Difficult to find factual/reliable information regarding the distance of the jump, speed, or information regarding the design of the ramp. From the video, a student may determine the angle of the ramp is approximately 67◦. However, this is unreasonable given the other constraints and the angle at which the video was taken. Other research shows motorcycles can reach a top speed of 115 mph, but there was not enough runway to reach this speed prior to reaching the ramp. The mostly likely distance provided by the Wikipedia of List of Evel Knievel Career Jumps is 129 feet for 19 cars. Solutions: Most likely the motorcycle reached a speed between 44 and 55 mph before reaching the end of the ramp. At 44 mph the ramp would have to have an angle of 41.7◦ and at 55 mph the ramp would have an angle of 19.74◦. Most likely the ramp was closer to 41.7◦. All other solutions within this range are acceptable.

Scenario 3: Serena Williams has a maximum serve of 122 mph with an average of 104.5 mph. A football field is 100 yards from end zone to end zone. However, from end line to end line it is 120 yards.

* Serena hits her average serve of 104.5 mph for 100 yards, she would need to serve at a 12.13◦ angle.
* Serena hits her average serve of 104.5 mph for 120 yards, she would need to serve at a 14.77◦ angle.
* Serena hits her max serve of 122 mph for 100 yards, she would need to serve at an 8.77◦ angle.
* Serena hits her max serve of 122 mph for 120 yards, she would need to serve at a 10.60◦ angle.

Scenario 4: Giants pitcher Matt Cain has a fastball that averages between 90 and 94 mph. The center field wall is 400 feet from home plate. Technically the grass begins the outfield, therefore the closest distance is approximately 155.5 ft.

* Matt throws at 90 mph 155.5 ft, he would need to release at an 8.34◦ angle.
* Matt throws at 94 mph 155.5 ft, he would need to release at a 7.63◦ angle.
* Matt throws at 90 mph 400 ft, he would need to release at a 23.8◦ angle.
* Matt throws at 94 mph 400 ft, he would need to release at a 21.3◦ angle.

Scenario 5: The distance is between 550 and 600 yards. The average speed of an arrow shot from a recursive bow (as used in the Hunger Games) is between 195 and 294.2 ft/sec depending on the size and weight of the arrow. The minimum speed for this distance is 241 mph.

* Arrow shot at 241 ft/sec for 550 yards would need to be shot at an angle of 33.03 degrees.
* Arrow shot at 241 ft/sec for 600 yards would need to be shot at an angle of 42.78 degrees.
* Arrow shot at 294.2 ft/sec for 550 yards would need to be shot at an angle of 18.94 degrees.
* Arrow shot at 294.2 ft/sec for 600 yards would need to be shot at an angle of 21.03 degrees.