**Subject area/course**: Mathematics/Algebra 2 or Integrated Math

**Grade level/band**: 10-12

**Task source**: Stanford Center for Assessment, Learning, and Equity (SCALE); authors: Kari Kokka and Vinci Daro

**Stopping Distance**

**TEACHER'S GUIDE**

1. **Task overview**:

This task begins with an animated video clip showing a cat jumping into the road, and a car nearly hitting it, which motivates a framing question: Can the driver stop in time? This sets the context for a whole-class brainstorm about what quantities might need to be considered in order to answer the question. The brainstorm creates some common ground for students to develop an approach to a well-specified problem, which is to figure out if they would be able to stop in time, given certain parameters and data about the situation. Students work on the problem individually first and then in pairs or small groups. The central mathematics of the problem includes figuring out that there is a linear relationship between time and distance that can be modeled with the given speed and ‘reaction time’, and then figuring out how to model the more complicated relationship between speed and distance, given a table of data; this can be approached in several mathematically valid ways.

While students work on the problem, the teacher circulates to listen for student comments and ideas that offer partial insights or suggest productive approaches or questions about the relationship between speed and distance; the teacher records selected comments/ideas on a ‘Student Ideas’ sheet in preparation for a whole-class discussion. The teacher then facilitates a discussion organized around the student comments/ideas they have recorded. The purpose of this discussion is to give students a chance to refine and develop their own ideas and their peers’ ideas.

Following work on the problem and the discussion about students’ ideas, students then consider three ideas from fictitious students about the table of data given in the problem. They choose from among these three ideas and any ideas from classmates shared by the teacher, and correct and develop 1-2 of these ideas into coherent arguments. This is individual written work that should be assessed formatively. Students will use feedback from the teacher on their work to inform their work on the culminating product.

Students then conduct research online about stopping distance and engage in a discussion about how a given formula relates to diagrams/graphs from their research. This discussion provides an opportunity to coordinate across different representations of the situation, with attention to representations of linear and quadratic relationships.

The culminating individual product is to write a recommendation about guidelines for ‘following distance’. Students first analyze the guidelines published by the California Department of Motor Vehicles (DMV) and the New York DMV, which are inconsistent, and use the mathematics of stopping distance to justify their recommendation about which state’s guidelines are more appropriate. They are provided three scenarios to consider as they develop their recommendation.

1. **Aligned standards:**
2. **Primary Common Core State Standards**

[CCSS.MATH.CONTENT.HSF.IF.B.4](http://www.corestandards.org/Math/Content/HSF/IF/B/4/) For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.

[CCSS.MATH.CONTENT.HSF.BF.A.1](http://www.corestandards.org/Math/Content/HSF/BF/A/1/) Write a function that describes a relationship between two quantities.

[CCSS.MATH.CONTENT.HSA.SSE.A.1](http://www.corestandards.org/Math/Content/HSA/SSE/A/1/) Interpret expressions that represent a quantity in terms of its context.

#### [CCSS.MATH.CONTENT.HSN.Q.A](http://www.corestandards.org/Math/Content/HSN/Q/) Reason quantitatively and use units to solve problems.

[CCSS.MATH.PRACTICE.MP2](http://www.corestandards.org/Math/Practice/MP2/) Reason abstractly and quantitatively.

[CCSS.MATH.PRACTICE.MP3](http://www.corestandards.org/Math/Practice/MP3/) Construct viable arguments and critique the reasoning of others.

[CCSS.MATH.PRACTICE.MP4](http://www.corestandards.org/Math/Practice/MP4/) Model with mathematics.

1. **Secondary Common Core State Standards**

[CCSS.MATH.CONTENT.HSA.CED.A.3](http://www.corestandards.org/Math/Content/HSA/CED/A/3/) Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

[CCSS.MATH.PRACTICE.MP1](http://www.corestandards.org/Math/Practice/MP1/) Make sense of problems and persevere in solving them.

[CCSS.MATH.PRACTICE.MP5](http://www.corestandards.org/Math/Practice/MP5/) Use appropriate tools strategically.

[CCSS.MATH.PRACTICE.MP6](http://www.corestandards.org/Math/Practice/MP6/) Attend to precision.

[CCSS.MATH.PRACTICE.MP8](http://www.corestandards.org/Math/Practice/MP8/) Look for and express regularity in repeated reasoning.

1. **Critical abilities**

Analysis of Information:Integrate and synthesize multiple sources of information (e.g., texts, experiments, simulations) presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to address a question, make informed decisions, understand a process, phenomenon, or concept, and solve problems while evaluating the credibility and accuracy of each source and noting any discrepancies among the data.

Communication in Many Forms:Use oral and written communication skills to learn, evaluate, and express ideas for a range of tasks, purposes, and audiences. Develop and strengthen writing as needed by planning, revising, editing, and rewriting while considering the audience.

Use of Technology:Present information, findings, and supporting evidence, making strategic use of digital media and visual displays to enhance understanding. Use technology, including the Internet, to research, produce, publish, and update individual or shared products in response to ongoing feedback, including new arguments or information.

Interpersonal Interaction and Collaboration:Develop a range of interpersonal skills, including the ability to work with others, to participate effectively in a range of conversations and collaborations.

Modeling, Design, and Problem Solving:Use quantitative reasoning to solve problems arising in everyday life, society, and the workplace, e.g., to plan a school event or analyze a problem in the community, to solve a design problem or to examine relationships among quantities of interest. Plan solution pathways, monitoring and evaluating progress and changing course if necessary, and find relevant external resources, such as experimental and modeling tools, to solve problems. Interpret and evaluate results in the context of the situation and improve the model or design as needed.

1. **Time/schedule requirements:**

This task will take approximately 1-2 weeks. See the outline below for a suggested implementation timeline.

* Part A: The Situation and The Problem – 2-3 days
* Part B: Building on Others’ Ideas – 1-2 days
* Part C: Research and Discussion – 1-2 days
* Part D: Culminating Product – 2-3 days

1. **Materials/resources:**

* Video of Tibbles the Cat: https://www.youtube.com/watch?v=R3yFku2L6V4
* Handout 1: The Situation (Initial Individual Notes)
* Handout 2A: The Problem (Initial Individual Work)
* Handout 2B: The Problem (Pair Work)
* Handout 3: Making Sense of Others’ Ideas (Pair Work)
* Handout 4: Building on Others’ Ideas (Individual Writing)
* Handout 5: Resource Card (Reference)
* Handout 6: Culminating Product (Individual Writing)
* Handout 7:Following Distance Guidelines for NY and CA (Reference)
* Handout 8: Three Scenarios to Consider (Reference)
* *Student Ideas* Discussion Tool (Teacher tool; optional for students)

1. **Prior knowledge:**

Students must have a solid understanding of using functions to model real world phenomena, and some experience using quadratic functions to model real world phenomena. Students must have experience graphing linear, quadratic, and exponential functions.

1. **Connection to curriculum:**

None listed.

1. **Teacher instructions:**

Part A: The Situation and The Problem [Whole Class, Individual, and Pair Work: 2-3 days]

Watch video of Tibbles the Cat [Whole class: 5 mins]

* Play first 32 seconds of video: https://www.youtube.com/watch?v=R3yFku2L6V4

Handout 1: The Situation [Individual or Pairs: 5-10 mins]

* Provide Handout 1 for students to make some initial notes on about what they would need to know in order to begin to develop an approach to the problem.
* Students are not to start solving the problem, but to just brainstorm the quantities that might be important to consider.

Discussion [Whole class: 5-10 mins]

* Students share ideas about what quantities might be important to consider in the situation.
* Record students’ ideas on the board, and as ideas accumulate, do some tentative grouping into categories: related to the driver (human), related to the vehicle, and related to driving conditions. Keep a record of these ideas for reference in Part D.

Handouts 2A & 2B: The Problem [Individual: 10 mins, Pairs/Groups: 15 mins]

* Provide Handout 2A to each student, and ask them to get started on the problem on their own for 10 minutes.
* Provide Handout 2B to each pair of students, and ask them to continue working on the problem together, adding to their individual work on Handout 2A as needed.

*Student Ideas* Discussion Tool [Use as students work on Handouts 2A and 2B]

* Circulate and notice patterns in students’ approaches as they work. Use a blank *Student Ideas* sheet (discussion tool with speech/thought bubbles) to record students’ comments and questions.
* Look for: Initial evidence of understanding the concept of a function, understanding the concept of covariation, use of variables, use of coordinate axes, attempts to determine type of function, recognizing possibilities of linear vs. quadratic vs. exponential functions and behaviors of each.
* Look for: Examples of students exhibiting math practices, especially SMP 2, SMP 4, and SMP 5.
* Listen for: Partial insights, initial steps forward, and common approaches. Record/paraphrase comments from students on *Student Ideas* discussion tool.
* Review individual work and prepare feedback questions and prompts.

*Suggested feedback questions and prompts for use during individual and pair work*

1. What types of function did you consider? How did you choose your function?

2. How do you know that it is the best model for the situation? Show two ways of justifying your choice.

3. How might you need to revise your model? What are the limitations of your model?

Discussion [Whole class: 10-20 mins]

* Facilitate a discussion about different approaches to modeling the relationship between speed and braking distance, in particular, and different ways of accounting for all of the given parameters in the situation as a whole.
* Use student ideas you have recorded on the *Student Ideas* discussion tool to call attention to particular approaches or partial insights that shed light on common issues in student work.
* Invite students to elaborate on their own ideas when presented for discussion, and to answer questions from classmates about their reasoning.

Part B: Building on Others’ Ideas [Pair & Individual Work: 1-2 days]

Handout 3: Making Sense of Others’ Ideas [Individual: 5-10 mins, Pairs: 10-20 mins]

* Provide Handout 3 for some initial individual think time for students to begin to make sense of each student’s idea, and then discuss the ideas in pairs and make some notes.

Handout 4: Building on Others’ Ideas [Individual: 20-30 mins]

* Provide Handout 4 for students to develop 1-2 ideas into more coherent arguments.
* Students can choose from among the ideas on Handout 3 as well as any ideas you have shared for discussion in Part A (using *Student Ideas* tool with speech/thought bubbles).

Part C: Research and Discussion [Group Work: 1-2 days]

Conduct research on stopping distance [Whole Class/Groups/Pairs: 10-20 mins]

* Students look up ‘stopping distance’ online and begin to interpret the graphs and diagrams that come up when clicking ‘Images’.

Handout 5: Resource Card [Whole Class/Groups: 30-60 mins]

* Use Handout 5 as a focus for discussion, and as a primary reference if online research is not possible (or introduces too much for students to process).
* Refer to Figures 1 and 2 on Handout 5 to focus a class discussion about how the two terms of the formula relate to the parts of the diagram. Ensure that students are attending to how the linear term and the quadratic term are represented in the diagram.
* You may want to have students work in groups to do the same exercise of relating the formula to a diagram with a different diagram from their online research.
* The purpose of this discussion is to give students an opportunity to coordinate across multiple representations of stopping distance, with a focus on how the linear term and the quadratic term are each represented.

Part D: Culminating Product [Individual Work: 2-3 days]

Discussion [Whole class: 10-20 mins]

* Facilitate a discussion about what quantities might matter for developing guidelines for following distance.

*Suggested discussion prompts:*

1. What quantities might matter for determining a safe distance between cars on the road? [Refer back to ideas from opening discussion for Part A as much as possible.]
2. What are some possible benefits of a shorter following distance? [From a traffic management perspective, a shorter following distance allows more cars to fit on the road at once]
3. How far do you travel in two seconds if you are traveling 1 mph? How far do you travel in three seconds if you are traveling 1 mph? What about if you are traveling 10 mph?
4. Do the type of car and tires matter? Do road conditions matter? In the formula on Handout 5, these were accounted for in the “friction coefficient,” *μ*.
5. What about the time it takes for a driver to apply the brakes once they realize then need to stop or slow down (In the formula on Handout 5, this was represented by *tp-r*, the “perception-reaction time”)? Does this vary depending on the age of the driver, or depending on whether the driver is tired or distracted?

Handout 6: Culminating Product

* Provide Handout 6 for students to read the description of what is expected.
* Individually, students compose a recommendation that synthesizes the relevant mathematics from all work on the task so far. Their recommendation should consist of a written argument and explanation supported by graphs, formulas, diagrams, and calculations.

Handout 7: Following Distance Guidelines from CA and NY

* Provide Handout 7 for students to review the guidelines published by the New York and California Departments of Motor Vehicles. (Online research is also an option, to extend comparison to other states’ guidelines.)

Handout 8: Three Scenarios to Consider (Optional)

* Provide Handout 8 to support further thinking about the mathematics of the situation as students begin to develop their recommendations.
* The purpose of these scenarios is to provide a structure for students to investigate the effects of different parameters and variables: speed, following distance, friction coefficient, and perception-reaction time.
* This handout should be considered optional; some students may perform better when relying on their own approach to the mathematics of the situation.

1. **Student support:**

As students work in pairs and groups on Handout 2B, provide each pair/group with a copy of the *Student Ideas* sheet (empty speech/thought bubbles) for their own use. This provides students with an optional tool for recording comments/questions from their group-mates and/or their own ideas. Instruct them to use the sheet as a listening tool for hearing things that make sense to them (or a thinking tool to record their own ideas) as they work together on the problem. Tell students you will collect any sheets students want to submit, and may use them to help focus a whole-class discussion about the relationship shown in the table. Then collect and review any sheets students want to submit, and present these to the class during the closing discussion for Part A to motivate the development of students’ thinking as they head into Part B.

1. **Extensions or variations:**

Ask students to explore the consequences of using a linear, quadratic, and exponential function to model the relationship between speed and distance using the braking distance data in the table or using data on braking distance for other vehicles found through research. Make a prediction first: which model will yield the greatest braking distance for a speed of 25mph? 65 mph?

1. **Scoring:**

Student work can be scored using the SCALE Math Performance Assessment Rubric (Grades 9-12).