Limits Lab: Exploring Limits Algebraically and Graphically

Part One: Evaluating Limits Algebraically

Directions – Evaluate the limits below using algebraic techniques. Please show all of your work.

1. $\lim_{p\to 4}\left[\dfrac{p^3-64}{4-p}\right]$	2. $\lim_{x\to \frac{1}{3}}\left[\dfrac{3x^2-7x+2}{-6x^2+5x-1}\right]$
3. $\lim_{s\to 9}\left[\dfrac{9-s}{\sqrt{s}-3}\right]$	4. $\lim_{x\to 0}\left[\dfrac{1-\cos x}{\sin x}\right]$
5. $\lim_{x\to -2}\left[\dfrac{\sqrt{2x+5}-1}{x+2}\right]$	6. $\lim_{x\to 0}\left[\dfrac{\frac{1}{x+4}-\frac{1}{4}}{x}\right]$

Part Two: Evaluating Limits at Infinity

Directions – Show all of your work in determining the following limits at infinity.

7. $\lim_{x\to-\infty}\left[\frac{5x^3-x^2+1}{10x^2-9x^3+4}\right]$	8. $\lim_{x\to-\infty}\left[\frac{x}{x^2-1}\right]$	9. $\lim_{x\to\infty}\left[\frac{3x-2}{\sqrt{2x^2+1}}\right]$
10. $\lim_{x\to-\infty}\left[\frac{45x^2+13x-18}{4x-9x^3}\right]$	11. $\lim_{x\to\infty}\left[\frac{\sqrt{x}-7}{6-5\sqrt{x}}\right]$	12. $\lim_{x\to\infty}\left[\frac{\sqrt[3]{x^3-8}}{2x}\right]$

Part Three: Applying Properties of Limits

Directions – Determine the values of each of the following limits using properties of limits.

$$\lim_{x\to4} f(x) = 5 \quad and \quad \lim_{x\to4} g(x) = -2$$

13. $\lim_{x\to4}[f(x)g(x)]$	14. $\lim_{x\to4}[xf(x)]$
15. $\lim_{x\to4}[f(x)+3g(x)]$	16. $\lim_{x\to4}\left[\left(g(x)\right)^3\right]$
17. $\lim_{x\to4}\left[\frac{f(x)}{f(x)-g(x)}\right]$	18. $\lim_{x\to4}\left[\frac{g(x)}{f(x)-1}\right]$

Part Four: Limits and Continuity of Piecewise Defined Functions

Directions – Determine the following limit and function values.

19.

$$h(t) = \begin{cases} 3t - 1, & t > 2 \\ -5, & t = 2 \\ 1 + 2t, & t < 2 \end{cases}$$

A) $\lim_{t \to 2^-} h(t) =$	B) $\lim_{t \to 2^+} h(t) =$
C) $\lim_{t \to 2} h(t) =$	D) $h(2) =$

20.

$$f(s) = \begin{cases} -s^2 - 4s - 2, & s \leq -2 \\ s^2 + 4s + 6, & s > -2 \end{cases}$$

A) $\lim_{s \to -2^-} f(s) =$	B) $\lim_{s \to -2^+} f(s) =$
C) $\lim_{s \to -2} f(s) =$	D) $f(-2) =$

21. Consider the function below:

$$f(x) = \begin{cases} \dfrac{x^2 + 4x - 32}{x^2 - 2x - 8}, & x \neq -2, 4 \\ 8, & x = 4 \end{cases}$$

Which of the following statements about f are true (circle all that apply)?

$I.$ f is not continuous at $x = 4$.

$II.$ $\lim_{x \to \infty} f(x) = 4$

$III.$ $x = 4$ is a vertical asymptote of the graph of $y = f(x)$

22. Find the value of m for which the function $h(x)$ below is continuous at $x = 2$.

$$h(x) = \begin{cases} 5x - 13, & x \leq 2 \\ x^2 - 7x + m, & x > 2 \end{cases}$$

23. Let a and b represent real numbers. Define $f(x)$ as follows:

$$f(x) = \begin{cases} ax^2 + x - b, & x \leq 2 \\ ax + b, & 2 < x < 5 \\ 2ax - 7, & x \geq 5 \end{cases}$$

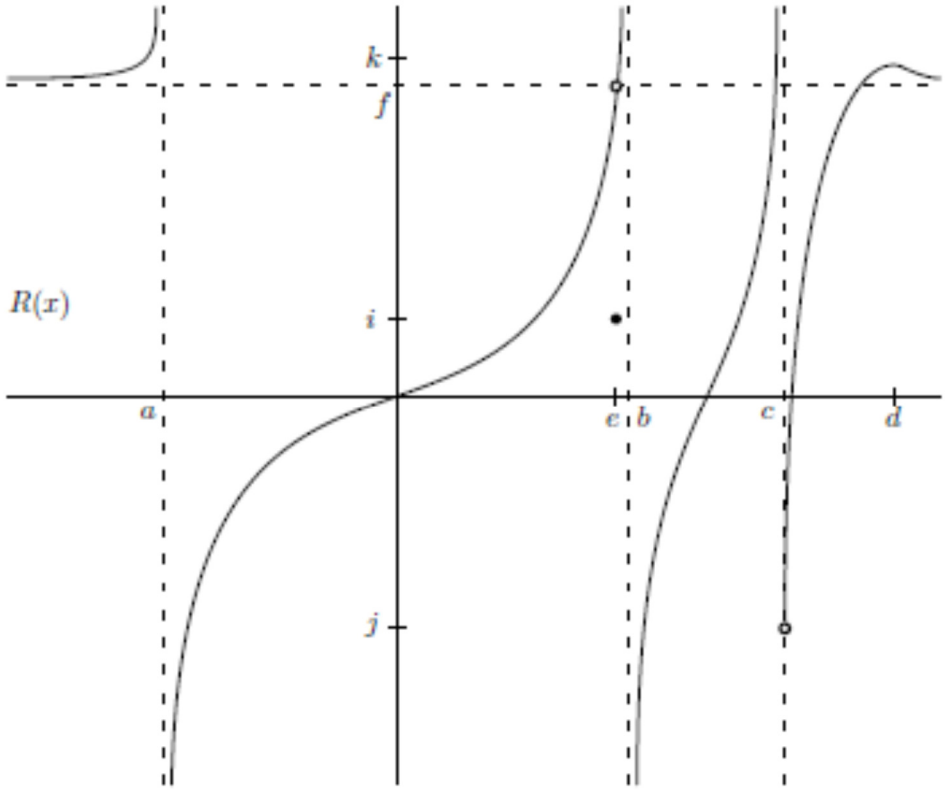
A) Find the values of a and b such that f is continuous everywhere.

B) Evaluate $\lim_{x \to 3} f(x)$.

C) Let $g(x) = \frac{f(x)}{x-1}$. Evaluate $\lim_{x \to 1} g(x)$.

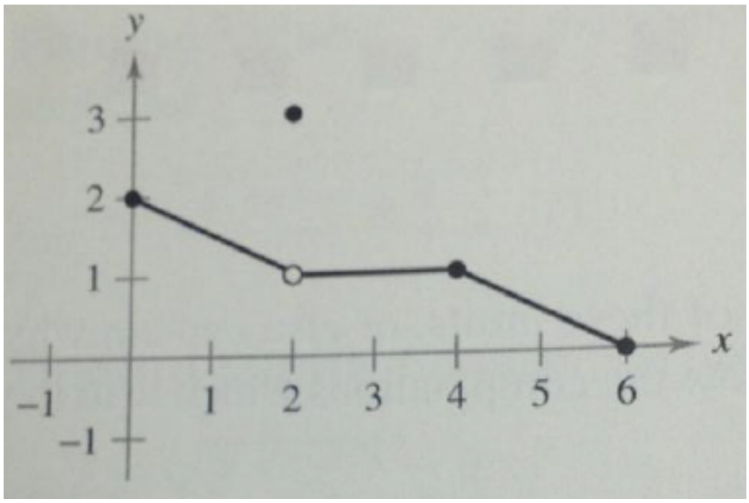
Part Five: Determining Limits Graphically

Directions – Determine the following limit and function values by analyzing the graph below.



24. $\lim_{x\to\infty} R(x) =$	25. $\lim_{x\to-\infty} R(x) =$	26. $\lim_{x\to a^+} R(x) =$	27. $\lim_{x\to a^-} R(x) =$
28. $\lim_{x\to a} R(x) =$	29. $\lim_{x\to b^+} R(x) =$	30. $\lim_{x\to b^-} R(x) =$	31. $\lim_{x\to b} R(x) =$
32. $\lim_{x\to 0} R(x) =$	33. $\lim_{x\to c} R(x) =$	34. $\lim_{x\to d} R(x) =$	35. $\lim_{x\to e} R(x) =$
36. $R(e) =$	37. $R(0) =$	38. $R(b) =$	39. $R(d) =$

40. The graph of function g is shown. Which of the following statements about g are true (circle all that apply)?

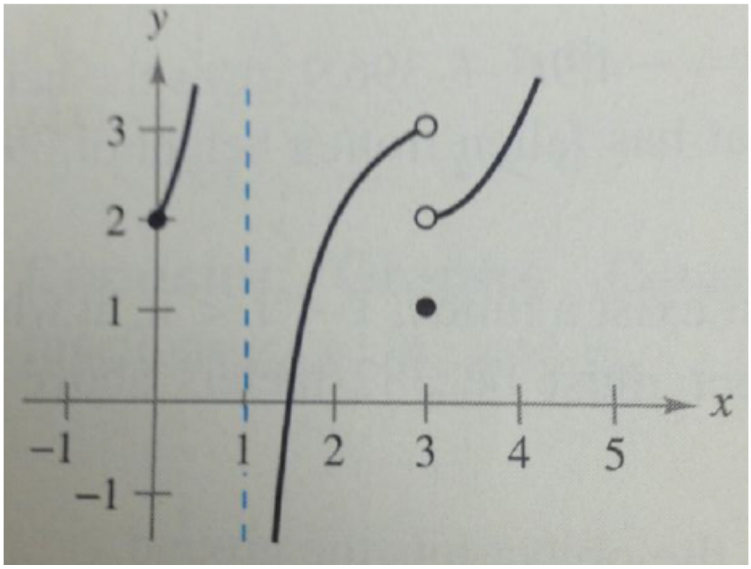


$I. \lim_{x \to 2} g(x) = 1$

$II. \lim_{x \to 2} g(x) = g(2)$

$III.$ $g(x)$ is continuous at $x = 3$

41. The graph of function f is shown. The line $x = 1$ is a vertical asymptote. Which of the following statements about f are true (circle all that apply)?



$I. \lim_{x \to 1} f(x) = \infty$

$II. \lim_{x \to 3} f(x) = 1$

$III. \lim_{x \to 3^-} f(x) < \lim_{x \to 3^+} f(x)$

$IV. \lim_{x \to 4} f(x)$ does not exist

$V. \lim_{x \to 0^+} f(x) = \lim_{x \to 3^+} f(x)$

Part Six: Analysis of Rational Algebraic Functions

42. The function $f(x)$ below is not continuous at three values of x. Find where $f(x)$ is discontinuous, and complete the chart that follows. *Show all of your work*.

$$f(x) = \frac{x^3 - 6x^2 + 11x - 6}{x^3 - 7x + 6}$$

Value of x at which $f(x)$ is discontinuous	**Type of discontinuity present** (circle your answer)	**Name a property of the graph** of $f(x)$ at this value of x
	Jump Removable Infinite	
	Jump Removable Infinite	
	Jump Removable Infinite	

43. The function f is defined as follows:

$$f(x) = \frac{x^2 + 5x + 6}{2x^2 + 7x + 3}$$

A) State the value(s) of x for which f is not continuous.

B) Evaluate $\lim_{x \to -3} f(x)$. Show the work that leads to your answer.

C) State the equation(s) for the vertical asymptote(s) for the graph of $y = f(x)$.

D) State the equation(s) for the horizontal asymptote(s) for the graph of $y = f(x)$.