2 Measures of Central Tendency

Overview and Connection to Culminating Project

Students work together to analyze data from a hypothetical class to help deepen their understanding of central tendency. They use reasoning to work backward from the mean in order to find information about data sets. They explore the effects of new data (especially outliers) on measures of central tendency. This deeper understanding will be necessary as they move forward with their Class Profile analysis.

Learning Objectives

Students will be able to

- Calculate mean and median.
- Identify mode.
- Describe, interpret, and draw conclusions from the central tendency of a data set.
- Describe what outliers are and how they affect measures of central tendency.
- Determine the ways in which measures change when new data is added to a set.
- Determine which statistics are best used to summarize different data sets.

Driving Question

- How can measures of central tendency be used to summarize data?

Assessment

Check for Understanding • How the Center Changes

Timeline

Lesson 1 • Central Tendency: Part I

Lesson 2 • Central Tendency: Part II

Lesson 3 • Ms. Garcia's Classes: Part I

Lesson 4 • Ms. Garcia's Classes: Part II

Check for Understanding • How the Center Changes

Materials, Supplies, and Technology

- Cheerios™ (or other small objects)
- Copies of Check for Understanding • How the Center Changes (see Handouts and Assessments)

Resources

- Stat Trek: Statistics and Probability Dictionary (mean)
- Stat Trek: Statistics and Probability Dictionary (median)
- Stat Trek: Statistics and Probability Dictionary (mode)
- Stat Trek: Statistics and Probability Dictionary (outlier)

LESSON 1

CENTRAL TENDENCY: PART I

WARM-UP

Find the Average

- Divide students into groups. Give each student a handful of 2–10 Cheerios™ (or other small objects). Students in the same group should have different amounts. The sum of Cheerios™ in a group should not be divisible by the number of students.
- Have them use the Cheerios™ to complete the warm-up activity in their Student Edition.
- Remind students to update their Sleep Log.

Your teacher will give you some Cheerios™.

1. Complete the table below to show how many Cheerios™ each person in your group has.

Person	Number of Cheerios™
Answers will vary.	

2. How many Cheerios™ would you say the **typical** student in your group has? Another way of thinking about this question is to say: I think most group members have about _____ Cheerios™.
   Answers will vary.

3. How did you determine your answer?
   Answers will vary.

4. How could you make it "fair" so that everyone in your group has the same number of Cheerios™?
   Answers will vary.

STUDENT EDITION

5. Were you able to divide the Cheerios™ evenly? If you cut the Cheerios™, how could you cut them so that everyone had the same number of Cheerios™?

   Answers will vary.

6. What are some things your group did particularly well to collaborate on this activity?

   Answers will vary.

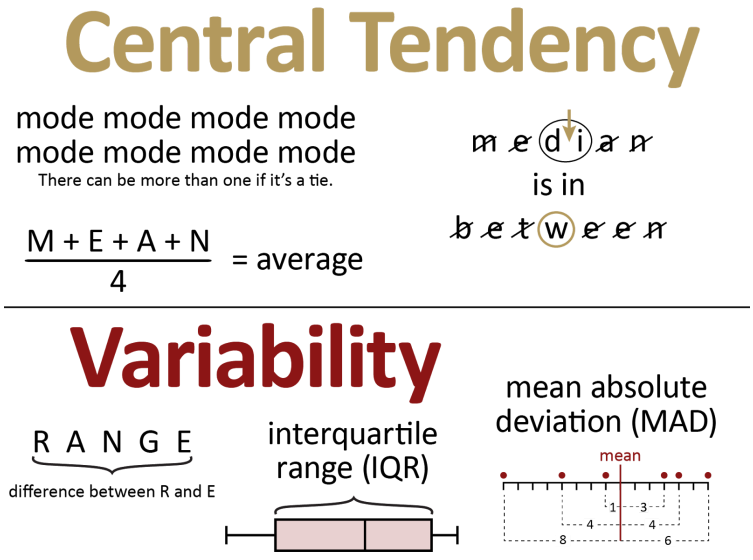
STUDENT EDITION

PROJECT ACTIVITY

Central Tendency

- Explain that one way statisticians can summarize large amounts of data (like in a Class Profile) is by describing the middle, or center, of the data.
- Have students look at the Central Tendency chart in their Student Edition.

1. Review the Central Tendency chart.



STUDENT EDITION

- Ask students to compare and contrast three ways of summarizing central tendency (mean, median, and mode) in a graphic organizer.

2. Compare and contrast three ways of summarizing central tendency (mean, median, and mode) using the graphic organizer below.

Mean	Median	Mode
All		

STUDENT EDITION

NOTE

Guide students to include information about how each measure is calculated, as well as what it is useful for.

- Have students return to their dot plot from the previous Learning Task and show how to find the mean, median, and mode for the data set.

3. Refer back to the dot plot you created in Learning Task 1, Lesson 2 (for the question in the "Who are We?" survey that you chose). Calculate the mean, median, and mode of the data set.

STUDENT EDITION

- At the end of class, have students share their mean, median, and mode and **explain which number they think is the most accurate representation of their data and why**. Have students update their graphic organizer as needed.
- Start a Statistics Word Wall that you can display, update, and reference throughout this unit. Add *mean*, *median*, and *mode* under the category of "Central Tendency."

Math Curricular Connections Suggestions

Engage NY: Median, Mode
NCTM Illuminations: Mean and Median Tool
Khan Academy: Mean and median

LESSON 2

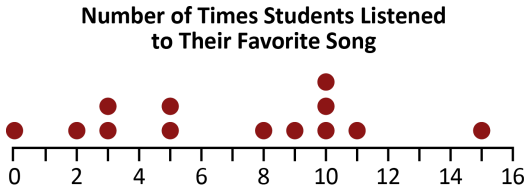
CENTRAL TENDENCY: PART II

WARM-UP

Dot Plot

- Have students work on the warm-up activity in their Student Edition.

1. The dot plot below shows how many times the students in a sixth grade class listened to their favorite song on Monday. Use the dot plot to answer the questions below.



a. What would be an appropriate label for the axis?
Number of times a favorite song was listened to

b. If a dot plot had a *y*-axis, what would be an appropriate label for it in this dot plot?
Number of students, or frequency

c. How many observations are reported in the data?
There are 13 observations.

d. What is the mode of the data?
The mode is 10.

e. What is the mean of the data?
The mean is 7.

f. What is the median of the data?
The median is 8.

g. Which measure of central tendency (mean, median, or mode) do you think is the best summary of the data? Why?
Accept any answer with a good rationale, such as:
The median, because it represents about the middle of the data, with 1 student 8 units lower than the median, 1 student 7 units higher than the median, and most students distributed fairly evenly around the median.

STUDENT EDITION

- Remind students to update their Sleep Log.

PROJECT ACTIVITY

Sleep and Central Tendency

- Create a data set of the number of hours of sleep students in your class got on a certain night. Have students use that data set to answer the questions in the Student Edition.

NOTE

If there is not an outlier in this data set, you might want to add your own to create a conversation about how outliers can skew the mean.

1. Create a dot plot for your class's sleep data. Make sure to include a title and to label the axis.

   Answers will vary.

2. Find the mean number of hours your class slept. Show your work.

   Answers will vary.

3. Find the median number of hours your class slept. Show your work.

   Answers will vary.

4. Why might the mean and median be different in this data set?

   Answers will vary.

5. Which of these central tendencies do you think is the best summary of this data? Why?

   Answers will vary.

6. Look at your mean hours of sleep. What are some ways that you could raise the mean by 15 minutes? Do you need to increase your sleep every night? What if you decrease your sleep some nights?

   Answers will vary.

STUDENT EDITION

- At the end of class, have students share their ideas about why the mean and median are different and which they think is a better representation (questions 4 and 5).
- Draw attention to the outlier(s). Use a tool like the NCTM Illuminations Mean and Median tool to show how outliers skew the mean.
- Have students update their central tendency graphic organizers from Lesson 1 as needed.

Math Curricular Connections Suggestions

Median, Mode (Engage NY or CPM)
NCTM Illuminations: Mean and Median tool
Khan Academy: Mean and Median

LESSON 3

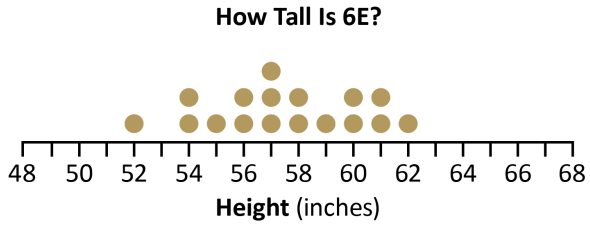
MS. GARCIA'S CLASSES: PART I

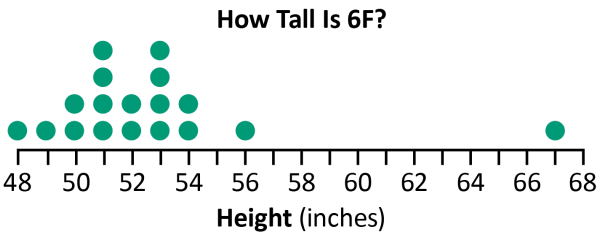
WARM-UP

Mean and Median

- Have students work on the warm-up activity in their Student Edition.

1. Look at the two data plots.





2. Decide whether it would be better to use the **mean** or **median** to summarize the data in each of these dot plots. Give reasons to support your claim.

In the first data plot, you could use the mean to summarize the data in the dot plot. Since there are no outliers, the mean and median should be fairly close. But I would choose the mean as the summary number because the data does not have any outliers.

In the second data plot,, I would use the median to summarize the data. The mean will be influenced by the outlier.

STUDENT EDITION

- Remind students to update their Sleep Log.

PROJECT ACTIVITY

Sleep Data from Ms. Garcia's Classes

- Have students begin working on the questions in the Student Edition about Ms. Garcia's classes. Students will have time in the next lesson to finish their work.

Answer the following questions. Make sure to show and explain all thinking. You will have time in the next lesson to finish your work.

1. Ms. Garcia's classes are also tracking their sleep. In her first-period class, 30 students have a mean of 8.25 hours of sleep on the night of September 20. How many total hours of sleep did her class get that night?

Show	**Explain**
$30 \cdot 8.25 = 247.5$ *The total number of hours of sleep for the whole class is 247.5 hours.*	*If the mean is 8.25, then each student averages 8.25 hours of sleep. So, if I multiply that by the number of students, I will get the total hours of sleep.*

2. One of the students in Ms. Garcia's class, Victor, spilled some juice on his Sleep Log.

Date	Hours of Sleep
1. 9/6	7.5 hours
2. 9/7	8 hours
3. 9/8	7 hours
4. 9/9	9 hours
5. 9/10	

He knows that the mean for the whole five-day week is 8 hours. How many hours of sleep did he get on the fifth night?

Show	**Explain**
$\frac{7.5+8+7+9+x}{5} = 8$ $7.5 + 8 + 7 + 9 + x = 40$ $x = 40 - 7.5 - 8 - 7 - 9$ $x = 8.5$	*I know the mean is 8, so all the numbers divided by 5 has to equal 8.*

STUDENT EDITION

3. There are 20 students in Ms. Garcia's second-period class. If you combine the data from this class with the data from her first-period class, the mean from question 1 changes. On September 20, the mean hours of sleep for **both her first- and second-period classes** is 8.75 hours.

   Find the mean hours of sleep on September 20 for **only the second-period class students**.

Show	**Explain**
$30 \cdot 8.25 + 20 \cdot x = 50 \cdot 8.75$ $247.5 + 20x = 437.5$ $20x = 190$ $x = 9.5$	*I multiply 30 • 8.25 to get the total hours in her first-period class. I multiply 50 • 8.75 to get the total hours in both classes. I can then subtract the total in the first-period class from the total in both classes to get the total in the second-period class. I divide by 20 to get the mean of the second-period class.*

4. Create a data set for Ms. Garcia's third-period class for September 20, where the mean is 9 hours and the median is 7 hours. What might the mean and median tell you about the students in the third-period class? Why are the mean and median so different?

Explain
The median is the middle and it is 7 hours. The mean is the average and it is 9 hours. Because the mean is higher, there may be high values that are influencing the mean. Maybe several students slept a lot of hours—probably over 12. Or, maybe most students slept 7 hours, without hardly anyone sleeping under 7 hours.

STUDENT EDITION

Math Curricular Connections Suggestions

NCTM Illuminations: Mean and Median tool
Khan Academy: Mean and median
Khan Academy: Mean and median challenge problems
Learnzillion: Data distributions
Learnzillion: Analyzing data

LESSON 4

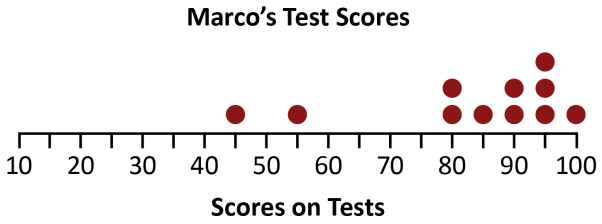
MS. GARCIA'S CLASSES: PART II

WARM-UP

Remove and Add Values

- Have students work on the warm-up activity in their Student Edition.

1. Use the dot plot below to answer the questions. Each dot represents the score for a different test. Show or explain all of your thinking.



a. What is the median test score?

The median is 90.

b. What would happen to the median test score if you removed the score of 55?

The median would stay the same.

c. What is the mean test score of the original data set?

The mean is 82.72.

d. What would happen to the mean test score if you removed the score of 55?

The mean would increase to 85.5.

e. Add one value to the data set below so that it has a median of 13. Show and explain your work.
{ 12, 9, 16, 21, 4}

You could add a data point of 16 to make the median 13. You would add 12 + 9 + 16 + 21 + 4 + 16 and then divide by 6 because there are 6 data points.

STUDENT EDITION

- Remind students to update their Sleep Log.

PROJECT ACTIVITY

Sleep Data from Ms. Garcia's Classes

- Continue to work on the problems about Ms. Garcia's class.
  See Lesson 3 for answers.

STUDENT EDITION

Math Curricular Connections Suggestions

Engage NY: Median, Mode
NCTM Illuminations: Mean and Median tool
Khan Academy: Mean and median
Khan Academy: Mean and median challenge problems
Learnzillion: Data distributions
Learnzillion: Analyzing data

CHECK FOR UNDERSTANDING

How the Center Changes

- Distribute the Learning Task 2 assessment—Check for Understanding • How the Center Changes.

A student in Ms. Garcia's class, Cece, got the following quiz scores: 87, 94, 82, 76, and 91. Next week she will take another quiz.

1. How will her quiz average change if she gets a score of 100 on the new quiz? Justify your answer.

   The mean will increase, because her average will be higher.
   Her average score now is: $\frac{87+94+82+76+91}{5} = 86$
   Her new average score will be: $\frac{87+94+82+76+91+100}{6} = 88.33...$

2. How will her quiz median change if she gets a score of 100 on the new quiz? Show and explain how you know.

   Her median score will increase.
   Her median score is now 87.
   After getting a score of 100, her median score will be 89.

3. What do you notice about the way Cece's quiz mean (average) and quiz median change when you add the score of 100?

   Both the mean and median increase.
   The median increases by more than the mean.

HANDOUTS AND ASSESSMENTS